Asset Embeddings

Xavier Gabaix   Ralph Koijen   Robert Richmond   Motohiro Yogo

Harvard - Chicago - NYU - Princeton

September 2024

Identifying similar firms

- In economics, we often try to find similar firms or assets.
  - E.g., similar growth rates, expected returns, risk, asset substitution, product markets, ...
- Common practice: Use observable characteristics.
  - E.g., industry definitions, accounting data, ...

Identifying similar firms

- In economics, we often try to find similar firms or assets.
  - E.g., similar growth rates, expected returns, risk, asset substitution, product markets, ...
- Common practice: Use observable characteristics.
  - E.g., industry definitions, accounting data, ...
- Those characteristics may be quite imperfect.
  - Standardized accounting data are an incomplete summary.
    - E.g., number of subscribers at Netflix, ...
  - New economic environments call for creative, new characteristics.
    - E.g., exposure to COVID-19, intangibles or AI.

Identifying similar firms

- In economics, we often try to find similar firms or assets.
  - E.g., similar growth rates, expected returns, risk, asset substitution, product markets, ...
- Common practice: Use observable characteristics.
  - E.g., industry definitions, accounting data, ...
- Those characteristics may be quite imperfect.
  - Standardized accounting data are an incomplete summary.
    - E.g., number of subscribers at Netflix, ...
  - New economic environments call for creative, new characteristics.
    - E.g., exposure to COVID-19, intangibles or AI.
- This paper: Use asset embeddings to measure firm similarity.

What are embeddings?

- Embeddings: Represent data (e.g., words) as vectors in a potentially high-dimensional space: $x_a \in \mathbb{R}^K$.
- Embeddings play a central role in the development of large language models (LLMs).
- In LLMs, embeddings capture the similarity between words and it allows us to do "math with words:

$$x_{\text{Paris}} - x_{\text{France}} + x_{\text{Spain}} \simeq x_{\text{Madrid}}.$$

What are embeddings?

- Embeddings: Represent data (e.g., words) as vectors in a potentially high-dimensional space: $x_a \in \mathbb{R}^K$.
- Embeddings play a central role in the development of large language models (LLMs).
- In LLMs, embeddings capture the similarity between words and it allows us to do “math with words:

$$x_{\text{Paris}} - x_{\text{France}} + x_{\text{Spain}} \simeq x_{\text{Madrid}}.$$

- Embedding vectors are learned from (lots of) data (not preselected).
- Despite the success of embedding techniques in these fields, their application in finance and economics largely unexplored.

Ideal data to estimate embeddings?

- We introduce the concept of asset embeddings.
  - A vector representation for each asset, that we learn from data.
- Which data to use?

Ideal data to estimate embeddings?

- We introduce the concept of asset embeddings.
  - A vector representation for each asset, that we learn from data.
- Which data to use?
- Our answer: Just like
  - documents organize words in language modeling,
  - images organize pixels in computer vision,
  - songs organize notes in audio,

  investors organize assets in finance and economics.

Ideal data to estimate embeddings?

- We introduce the concept of asset embeddings.
  - A vector representation for each asset, that we learn from data.
- Which data to use?
- Our answer: Just like
  - documents organize words in language modeling,
  - images organize pixels in computer vision,
  - songs organize notes in audio,

  investors organize assets in finance and economics.
- Theoretically, we show how embeddings can be recovered by "inverting the asset demand system."

Which method to learn embeddings?

- Which method to use?

Which method to learn embeddings?

- Which method to use?
- Traditional approach: LSA (Latent Semantic Analysis), which is related to PCA/recommender systems.
- The recent ML/AI literature went way beyond that.
  - Context-invariant embeddings: E.g., GloVe and Word2Vec.
  - Embeddings with context: E.g., transformer models (e.g., BERT and GPT).
  - Parameters are estimated using masked language modeling.

Investor embeddings

- Even though our focus is on asset embeddings, we obtain investor embeddings as a by-product: $\lambda_i \in \mathbb{R}^K$.
  - Learned vector representations of each investor's "taste for characteristics"

Investor embeddings

- Even though our focus is on asset embeddings, we obtain investor embeddings as a by-product: $\lambda_i \in \mathbb{R}^K$.
  - Learned vector representations of each investor's "taste for characteristics"
- Examples of applications:
  - Classify investors beyond institutional type, size, and activeness.
  - Identify crowded trades.
  - Performance measurement (extending Daniel, Grinblatt, Titman, and Wermers, 1997).

Five main contributions

1. Micro-found the use of holdings data as embeddings data (in the paper).
2. Three benchmarks to compare asset embedding models.
   - Building on the success of benchmark in AI (e.g., ImageNet).
3. Explore different modeling architectures to learn asset embeddings based on language models.
4. Evaluate benchmarks for asset embeddings, text-based embeddings, and observed characteristics.
5. Use earnings calls data to interpret the embeddings.
   - Extends to any other form of text data (e.g., WSJ articles, analyst reports, ...).

Recommender systems

- Recommender systems, with $\theta = (x_a, \lambda_i, \delta_i, \delta_a)$,

$$\min_{\theta} \frac{1}{IA} \sum_{i,a} (h_{ia} - \delta_i - \delta_a - \lambda_i' x_a)^2 + \frac{\xi}{IK} \sum_{i} \lambda_i' \lambda_i + \frac{\xi}{AK} \sum_{a} x_a' x_a,$$

  where
  - h_{ia}: Log holdings.
  - x_a: Asset embeddings.
  - λ_{iq}: Investor embeddings.
- Analogous to LSA in the NLP literature.[1]

[1] Dumais, Furnas, Landauer, and Deerwester (1988).

Implementations of recommender systems

- To understand how to best extract information from holdings, we consider five variants:

  1. Binary, $\mathbb{I}_{H_{ia}>0}$.
  2. Percentile ranks of H_{ia} with missing values set to zero.
  3. h_{ia} with missing values set to zero.
  4. h_{ia} with missing values set to the smallest active position.
  5. h_{ia} using only the non-missing values.

Word2Vec

- General approach to estimate language models, such as Word2Vec,[2]
  - Task: Guess masked words.
    - E.g. “Please pass me the _____ and pepper”.
  - Use a context window to maximize the probability of a missing word given the context info:

$$\mathbb{P}(w_a \mid w_c) = \frac{\exp(x_a' x_c)}{\sum_b \exp(x_b' x_c)}.$$

[2]Mikolov, Sutskever, Chen, Corrado, Dean (2013a, b).

Word2Vec

- General approach to estimate language models, such as Word2Vec,[2]
  - Task: Guess masked words.
    - E.g. "Please pass me the _____ and pepper".
  - Use a context window to maximize the probability of a missing word given the context info:

$$\mathbb{P}(w_a \mid w_c) = \frac{\exp(x_a' x_c)}{\sum_b \exp(x_b' x_c)}.$$

- Estimation using holdings data:
  - Sentences $\Rightarrow$ Investors.
  - Words $\Rightarrow$ Assets.
  - Objective: Guess masked assets (cross entropy).

[2] Mikolov, Sutskever, Chen, Corrado, Dean (2013a, b).

Masked asset modeling

- Example: The ARKK ETF in July 2023:

Holdings Data - ARKK
As of 07/07/2023

ARK INVEST

ARKK
ARK Innovation ETF

	Company	Ticker	CUSIP	Shares	Market Value ($)	Weight (%)
1	TESLA INC	**TSLA**	88160R101	3,496,872	$967,024,982.88	12.43%
2	COINBASE GLOBAL INC -CLASS A	**COIN**	19260Q107	7,945,138	$620,515,277.80	7.98%
3	ROKU INC	**ROKU**	77543R102	8,865,426	$546,110,241.60	7.02%
4	~~ZOOM VIDEO COMMUNICATIONS-A~~	~~**ZM**~~	~~98980L101~~	~~8,258,591~~	~~$534,248,251.79~~	~~6.87%~~
5	UIPATH INC - CLASS A	**PATH**	90364P105	28,152,366	$463,106,420.70	5.95%
6	BLOCK INC	**SQ**	852234103	7,069,493	$456,759,942.73	5.87%
7	EXACT SCIENCES CORP	**EXAS**	30063P105	4,031,264	$368,739,718.08	4.74%
8	UNITY SOFTWARE INC	**U**	91332U101	8,350,868	$338,627,697.40	4.35%
9	SHOPIFY INC - CLASS A	**SHOP**	82509L107	5,430,238	$335,751,615.54	4.32%
10	DRAFTKINGS INC-CL A	**DKNG UW**	26142V105	12,035,607	$303,658,364.61	3.90%

Context and self-attention: A simple example

- So far, we have one x_a per asset, say, Apple, with no context.
- How does attention[3] work?

[3] Vaswani, Shazeer, Parmar, Uszkoreit, Jones, Gomez, Kaiser, Polosukhin (2017).

Context and self-attention: A simple example

- So far, we have one x_a per asset, say, Apple, with no context.
- How does attention[3] work?

1. $\mathcal{H}_i$: Stocks in the portfolio of manager i.
2. For stock $a \in \mathcal{H}_i$, compute a similarity score with the other stocks $b \in \mathcal{H}_i$

$$\sigma_{ab} = x_a' x_b.$$

   x_a: Query.
   x_b: Key.
3. Compute the contextualized embedding, x_a^i,

$$x_a^i = \sum_{b \in \mathcal{N}_i} \frac{e^{\sigma_{ab}}}{\sum_{c \in \mathcal{N}_i} e^{\sigma_{ac}}} x_b.$$

   x_b: Value.

[3] Vaswani, Shazeer, Parmar, Uszkoreit, Jones, Gomez, Kaiser, Polosukhin (2017).

Generalizing attention: Transformers

- Transformer models generalize this idea.
  - Query: $q_a = W^Q x_a$.
  - Key: $k_a = W^K x_a$.
  - Value: $v_a = W^V x_a$.
- The contextualized embedding is then computed as

$$x_a^i = \sum_{b \in \mathcal{N}_i} \frac{e^{\sigma_{ab}}}{\sum_{c \in \mathcal{N}_i} e^{\sigma_{ac}}} v_b, \qquad \sigma_{ab} = q_a' k_b.$$

- The matrices W_Q, W_K, and W_V are learned from (lots of) data and determine which aspects of the context are important.

GENERALIZING ATTENTION: TRANSFORMERS

- Transformer models generalize this idea.
  - Query: $q_a = W^Q x_a$.
  - Key: $k_a = W^K x_a$.
  - Value: $v_a = W^V x_a$.
- The contextualized embedding is then computed as

$$x_a^i = \sum_{b \in \mathcal{N}_i} \frac{e^{\sigma_{ab}}}{\sum_{c \in \mathcal{N}_i} e^{\sigma_{ac}}} v_b, \qquad \sigma_{ab} = q_a' k_b.$$

- The matrices W_Q, W_K, and W_V are learned from (lots of) data and determine which aspects of the context are important.
- Features of the full AssetBERT model
  - Stack multiple attention layers with multi-headed attention.
  - Add a feed-forward layer between each self-attention layer:

$$FF(x) = \max(0, xW_1 + b_1)W_2 + b_2,$$

    where the dimensionality of the inner layer $\gg \dim(x)$.
  - Add position embeddings.

Data

- Holdings data from FactSet:
  - Hedge funds, mutual funds, ETFs, closed-end funds, variable annuity funds.
- Sample construction:
  - 2005.Q1 - 2022.Q4.
  - Remove nano and micro caps.
  - Keep investors (stocks) with at least 20 positions (investors).
- Accounting data and stock returns from CRSP / Compustat, using the Jensen, Kelly, and Pedersen (2023) construction.
- Earnings calls data from FactSet.

Three benchmarks

1. Predicting relative valuations.
   - Decompose $m_a = \beta_0 + \beta_1 b_{at} + m_a^{\perp}$.
   - Estimate $m_a^{\perp} = \gamma_0 + \gamma_1' x_a + \epsilon_a$ on 80% of the sample.
   - Evaluate using the R^2 on the remaining 20%.

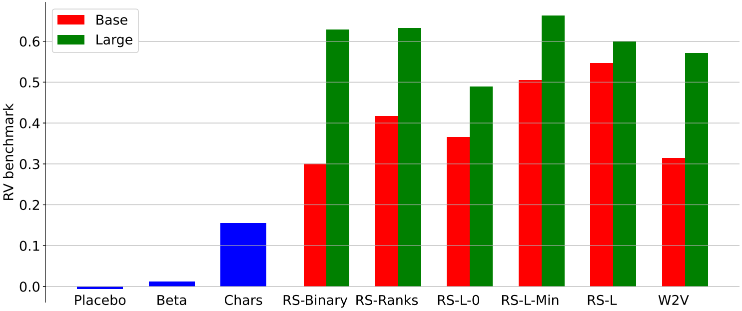
Three benchmarks

1. Predicting relative valuations.
   - Decompose $m_a = \beta_0 + \beta_1 b_{at} + m_a^{\perp}$.
   - Estimate $m_a^{\perp} = \gamma_0 + \gamma_1' x_a + \epsilon_a$ on 80% of the sample.
   - Evaluate using the R^2 on the remaining 20%.
2. Explaining comovement.
   - Estimate $r_{am} = c_m + x_{a,q-1}' f_m + \epsilon_{am}$ on 80% of the sample.
   - Evaluate using the R^2 on the remaining 20%.

Three benchmarks

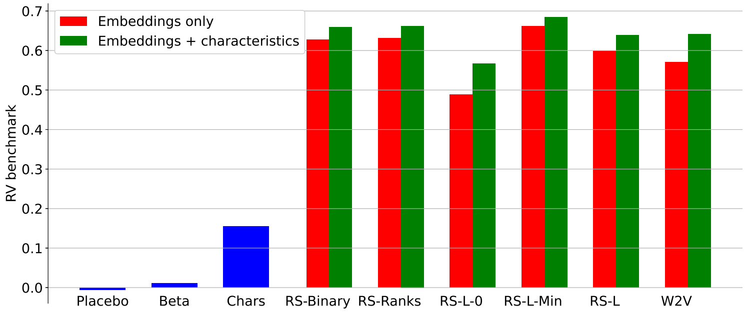
1. Predicting relative valuations.
   - Decompose $m_a = \beta_0 + \beta_1 b_{at} + m_a^{\perp}$.
   - Estimate $m_a^{\perp} = \gamma_0 + \gamma_1' x_a + \epsilon_a$ on 80% of the sample.
   - Evaluate using the R^2 on the remaining 20%.
2. Explaining comovement.
   - Estimate $r_{am} = c_m + x_{a,q-1}' f_m + \epsilon_{am}$ on 80% of the sample.
   - Evaluate using the R^2 on the remaining 20%.
3. Asset similarity in managed portfolios.
   - Mask the second position of a fund.
   - Estimate the probability of the identity of the second holding using embeddings/characteristics.

Benchmark 1: Predicting relative valuations



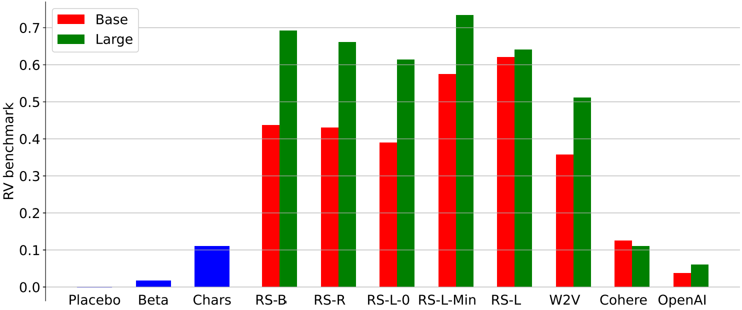
- Main takeaways:
  - Holdings-based asset embeddings perform well relative to characteristics.
  - High-dimensional models perform significantly better.

Combining embeddings and characteristics



- Main takeaway:
  - Adding characteristics to asset embeddings does not improve the benchmark much.

Text-based embeddings



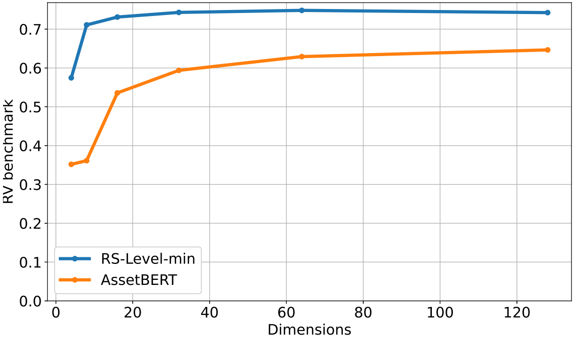
- Main takeaway:
  - Text-based asset embeddings do not perform well.

Understanding text-based embeddings

- Using OpenAI's text-based embeddings, we search for the most similar firms (using cosine similarity).
- OpenAI's embeddings mix economic and semantic similarity.

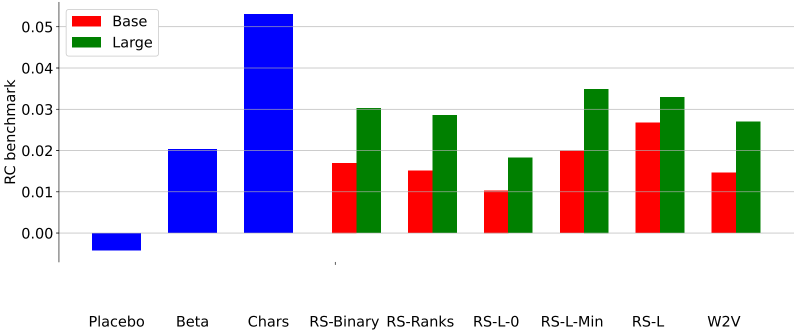
	Similar Firms as predicted by OpenAI		
Input company	Apple Inc	Citigroup Inc	Walmart Inc
Rank 1	Appian Corp	Citizens Financial Group Inc	Walgreens Boots
Rank 2	Adobe Inc	Goldman Sachs Group Inc	Home Depot Inc
Rank 3	Interdigital Inc	American International Group Inc	Murphy Usa Inc
Rank 4	Microsoft Corp	Comerica Inc	Amazon Com Inc
Rank 5	Gopro Inc	Cigna Corp New	Qurate Retail Inc
Rank 6	Netapp Inc	Capital One Financial Corp	Big Lots Inc
Rank 7	Intel Corp	Caci International Inc	Burlington Stores
Rank 8	Alphabet Inc	Capital City Bank Group	Dollar Tree Inc
Rank 9	Autodesk Inc	C N O Financial Group Inc	Nordstrom Inc
Rank 10	Appfolio Inc	Jpmorgan Chase & Co	Kohls Corp

High-dimensional embeddings

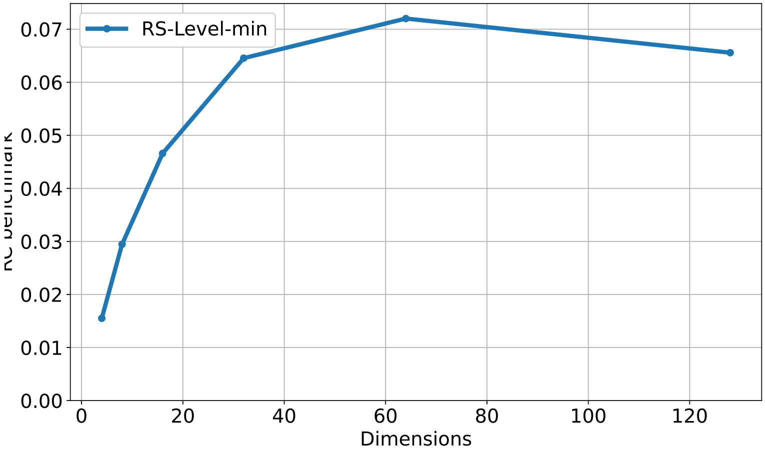


- Main takeaways:
  - High-dimensional models perform particularly well.
  - AssetBERT performs well, but outperformed by the simpler recommender system.

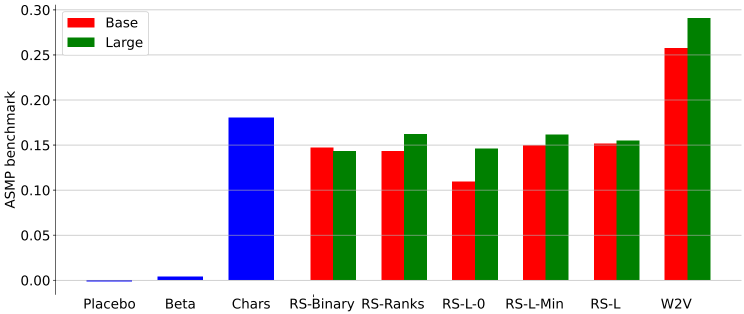
Benchmark 2: Explaining comovement



High-dimensional embeddings

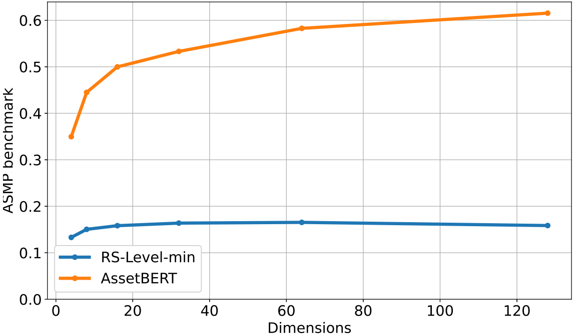


Benchmark 3: Asset similarity



- Main takeaway:
  - Word2Vec performs significantly better than recommender systems and observed characteristics.

High-dimensional embeddings



- Main takeaway:
  - AssetBERT performs better than the simpler recommender system and Word2Vec on this benchmark.

Investor similarity

- The simple recommender system already leads to a reasonable clustering of investors.

Fund	Dimensional US Large Cap Value ETF
Rank 1	SA US Value Fund
Rank 2	Dimensional Funds ICVC - International Value Fund
Rank 3	PGIM Quant Solutions Large-Cap Value Fund
Rank 4	UBS (Irl) ETF Plc - Factor MSCI USA Prime Value ESG UCITS ETF
Rank 5	Columbia Multi Manager Value Strategies Fund
Fund	iShares Exponential Technologies Index ETF
Rank 1	Multi Units LU - Lyxor MSCI Disruptive Tech. ESG Filtered
Rank 2	LUX IM - AI & DATA
Rank 3	AtonR Fund (The)
Rank 4	Dux Umbrella FI - Trimming USA Technology
Rank 5	HANetf ICAV - HAN-GINS Innovative Technologies UCITS ET
Fund	Virtus LifeSci Biotech Products ETF
Rank 1	Global X Genomics & Biotechnology ETF
Rank 2	BNY Mellon Global Fds. Plc - Smart Cures Innovation Fund
Rank 3	WisdomTree BioRevolution Fund
Rank 4	JPMorgan Funds - Thematics - Genetic Therapies
Rank 5	JSS Investmentfonds II - Sustainable Eq. - Future Health

Interpreting asset and investor embeddings

- Asset embeddings yield clusters of stocks.
- We use OpenAI's `GPT-4o` model to summarize the earnings calls of groups of firms and identify
  - Main common risks.
  - Main growth opportunities.
  - ...
- To avoid generic risks, we can add a group of firms (sampled across industries) as a reference point.
- The same logic applies to investor embeddings using, e.g., information in fund prospectuses, analyst reports, et cetera.

Conclusions

- Recent advances in AI/ML can be applied to economics and finance via asset embeddings.
- We provide a micro foundation for using holdings data.
- We adjust methods that have been successful in related areas (e.g., NLP, vision, ...) to economics:
  - Recommender systems, Word2Vec, transformer models.
- In progress:
  - Other asset classes: Fixed income.
    - Use embeddings to improve on ratings and distance to default to explain yields, yield volatility, and default.
    - An opportunity to redesign the architecture of fixed income markets.
  - Generate stress scenarios by simulating investor and asset embeddings, combined with an asset demand system (Koijen and Yogo, 2019).